Exercises

2.11 **Stocks, volatility, and diversification.**[1] (Finance, Computation) ②

Stock prices are fairly good approximations to random walks. The Standard and Poor's 500 index is a weighted average of the prices of five hundred large companies in the United States stock-market.

From the 'Stock market' link on the computer exercises web site [129], download SandPConstantDollars.dat and the hints files for your preferred programming language. Each line in the data file represents a weekday (no prices are listed on Saturday or Sunday). The first column is time t (in days, since mid-October 1982), and the second column is the Standard and Poor's index $SP(t)$ for that day, corrected for inflation (using the consumer price index for that month).

Are the random fluctuations in the stock-market due to external events?

(a) *Plot the price index versus time. Notice the large peak near year 2000. On September 11, 2001 the World Trade Center was attacked (day number 6903 in the list). Does it seem that the drop in the stock-market after 2000 is due mostly to this external event?*

Sometimes large fluctuations are due to external events; the fluctuations in ecological populations and species are also quite random, but the dinosaur extinction was surely caused by a meteor.

What do the steps look like in the random walk of Standard and Poor's index? This depends on how we define a step; do we ask how much it has changed after a year, a month, a week, or a day?

A technical question arises: do we measure time in days, or in trading days? We shall follow the finance community, and consider only trading days. So, we will define the lag variable ℓ to be one trading day for a daily percentage change (even if there is a weekend or holiday in between), five for a weekly percentage change, and 252 for a yearly percentage change (the number of trading days in a typical year).

(b) *Write a function P_ℓ that finds all pairs of time points from our data file separated by a time interval $\Delta t = \ell$ and returns a list of per cent changes*

$$P_\ell(t) = 100\frac{SP(t+\ell) - SP(t)}{SP(t)}$$

over that time interval. Plot a histogram of the daily changes, the weekly changes, and the yearly changes. Which of the three represents a reasonable time for you to stay invested in the Standard and Poor's index (during which you have mean percentage growth larger than a tiny fraction of the fluctuations)? Also, why do you think the yearly changes look so much more complicated than the other distributions? (Hint for the latter question: How many years are there in the data sample? Are the steps $SP(n) - SP(n-\ell)$ independent from $SP(m) - SP(m-\ell)$ for $n - m < \ell$? The fluctuations are determined not by the total number of steps, but by the effective number of independent steps in the random walk.)

The distributions you found in part (b) for the shorter lags should have looked quite close to Gaussian—corresponding nicely to our Green's function analysis of random walks, or more generally to the central limit theorem. Those in mathematical finance, though, are interested in the deviations from the expected behavior. They have noticed that the tails of the distribution deviate from the predicted Gaussian.

(c) *Show that the logarithm of a Gaussian is an inverted parabola. Plot the logarithm of the histogram of the weekly percentage changes from part (b). Are there more large percentage changes than expected from a Gaussian distribution (fat tails) or fewer?* (Hint: Far in the tails the number of measurements starts becoming sparse, fluctuating between zero and one. Focus on the region somewhat closer in to the center, where you have reasonable statistics.)

Some stocks, stock funds, or indices are more risky than others. This is not to say that one on average loses money on risky investments; indeed, they

[1] From *Statistical Mechanics: Entropy, Order Parameters, and Complexity* by James P. Sethna, copyright Oxford University Press, 2007, page 31. A pdf of the text is available at pages.physics.cornell.edu/sethna/StatMech/ (select the picture of the text). Hyperlinks from this exercise into the text will work if the latter PDF is downloaded into the same directory/folder as this PDF.

usually on average pay a better return than conservative investments. Risky stocks have a more variable return; they sometimes grow faster than anticipated but sometimes decline steeply. Risky stocks have a high standard deviation in their percentage return. In finance, the standard deviation of the percentage return is called the *volatility*

$$v_\ell = \sqrt{\left\langle \left(P_\ell(t) - \bar{P}_\ell\right)^2 \right\rangle}.$$

(d) *Calculate the daily volatility, the weekly volatility, and the monthly volatility of the inflation-corrected Standard and Poor's 500 data. Plot the volatility as a function of lag, and the volatility squared as a function of lag, for lags from zero to 100 days. Does it behave as a random walk should?*

The volatility of a stock is often calculated from the price fluctuations within a single day, but it is then *annualized* to estimate the fluctuations after a year, by multiplying by the square root of 252.

The individual stocks in the Standard and Poor's 500 index will mostly have significantly higher volatility than the index as a whole.

(e) *Suppose these five hundred stocks had mean annual percentage returns m_i and each had mean volatility σ_i. Suppose they were equally weighted in the index, and their fluctuations were uncorrelated. What would the return and volatility for the index be? Without inside information[2] or insight as to which stocks will have higher mean returns, is there any average disadvantage of buying portions of each stock over buying the index? Which has lower volatility?*

Investment advisers emphasize the importance of *diversification*. The fluctuations of different stocks are not independent, especially if they are in the same industry; one should have investments spread out between different sectors of the economy, and between stocks and bonds and other types of investments, in order to avoid risk and volatility.

[2] Insider trading is illegal.