Infectious disease dynamics

Colizza & Vespignani

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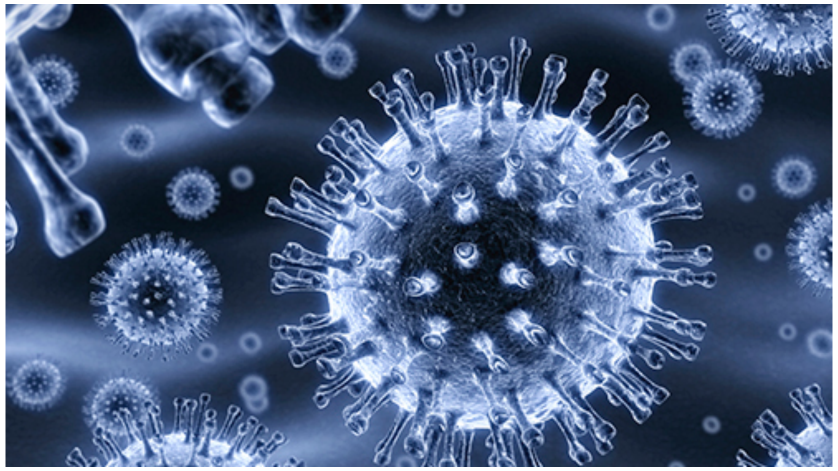

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Infection timeline

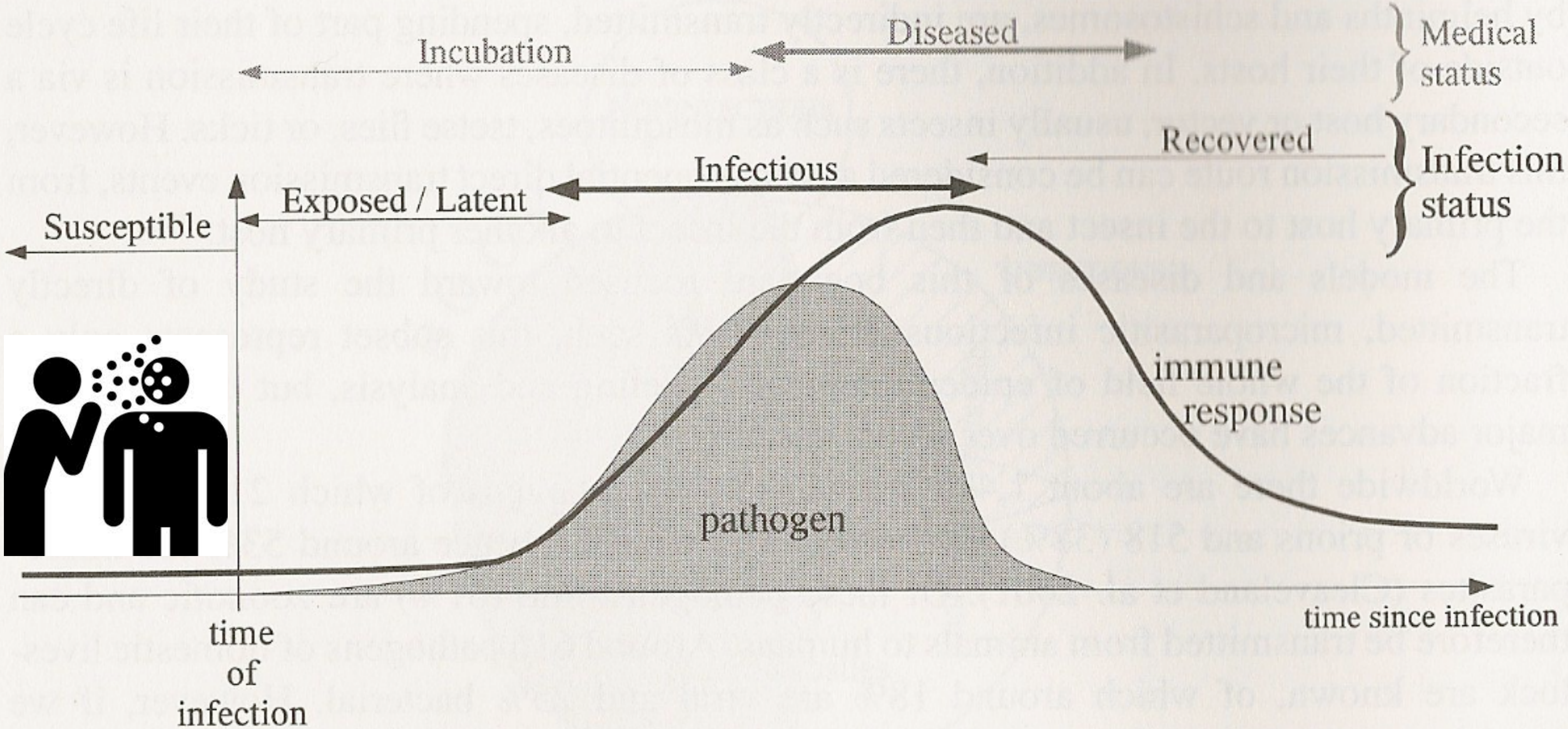
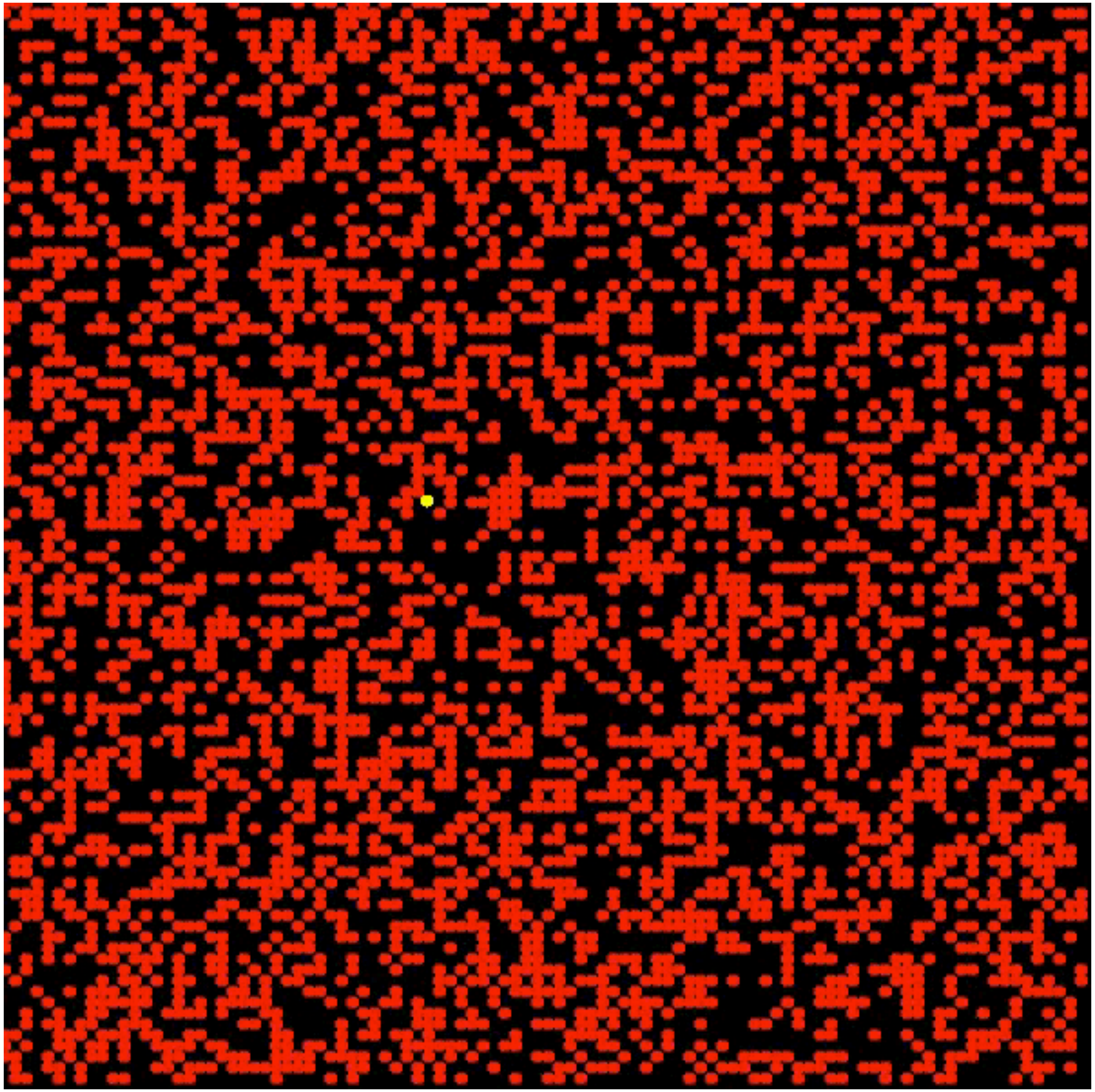


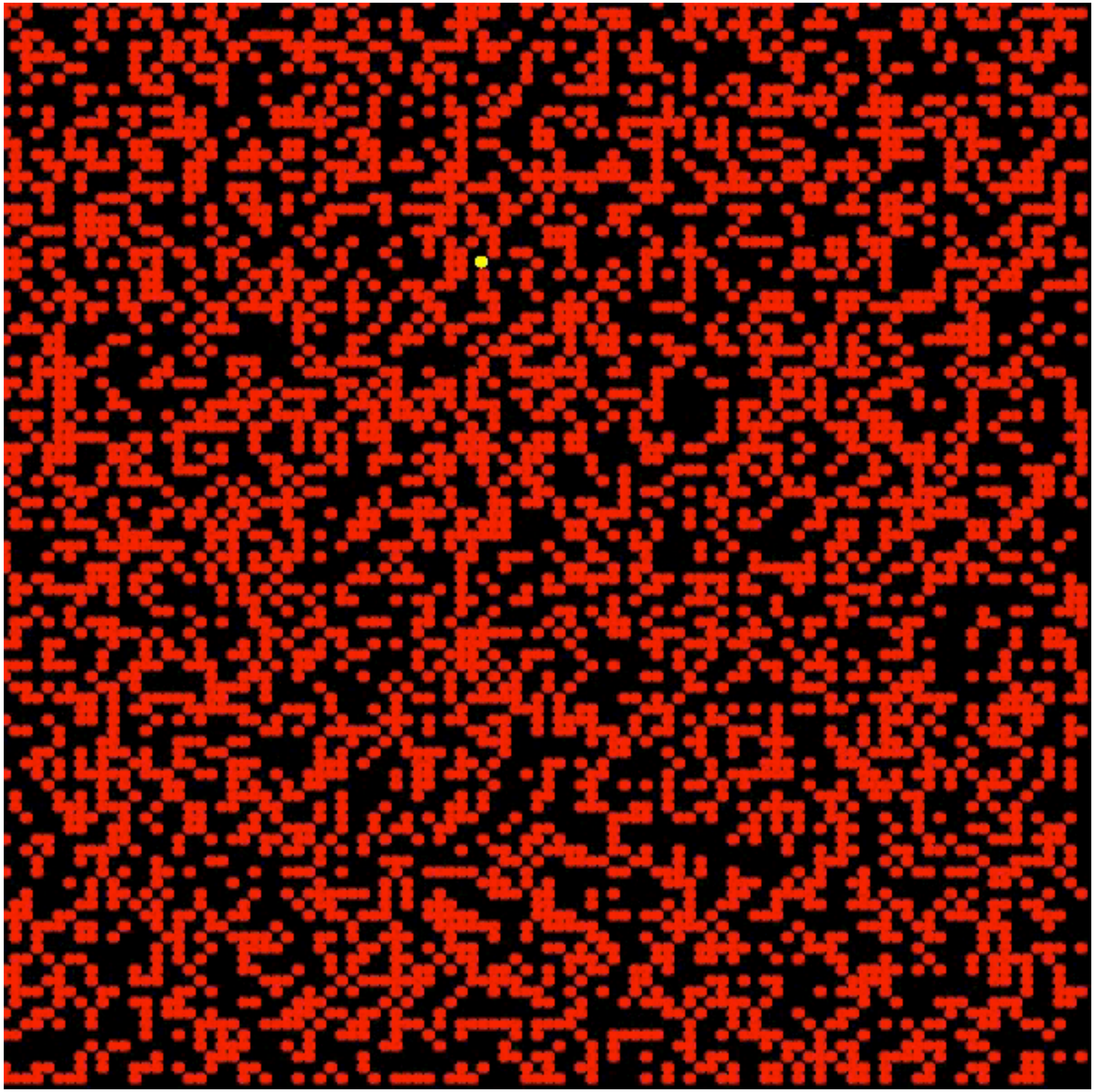
Figure 1.2. A caracature of the time-line of infection, showing the dynamics of the pathogen (gray area) and the host immune response (black line) as well as labeling the various infection classes: susceptible, exposed, infectious, and recovered. Note that the diseased period, when symptoms are experienced, is not necessarily correlated with any particular infection class.

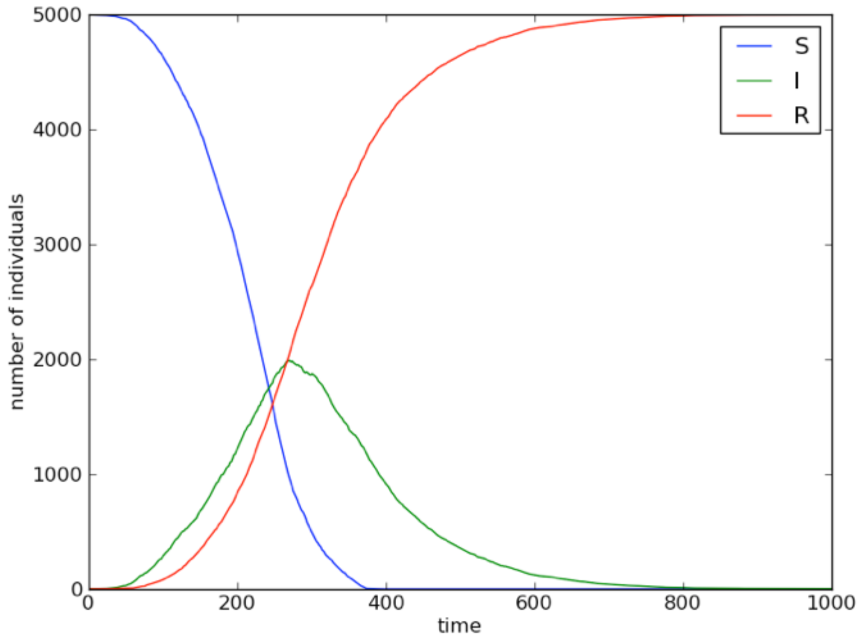
Keeling & Rohani, Fig. 1.2

The spread of disease



The spread of disease





Scales, foci & the multidisciplinary nature of infectious disease modeling & control

response

- control strategies
- epidemiology
- public health & logistics
- economic impacts

between hosts

- disease ecology
- demography
- vectors, water, etc.
- zoonoses
- weather & climate

↑ transmission

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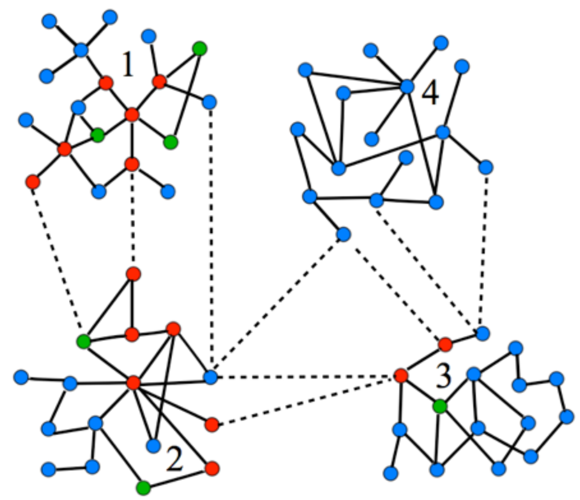
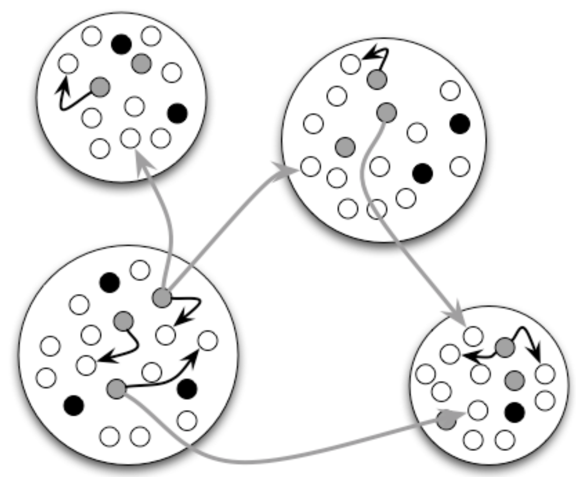
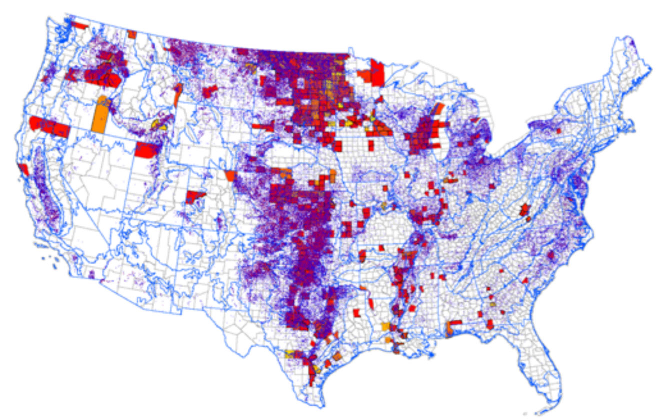
within-host

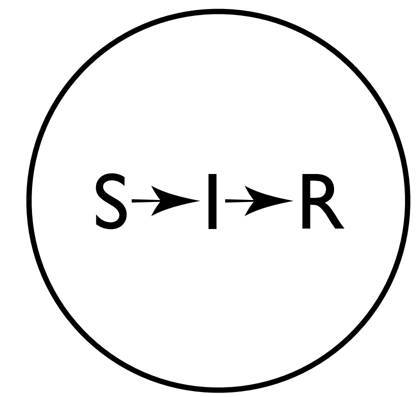
- virology, bacteriology, mycology, etc.
- immunology

Disease models at various levels of resolution









agent-based models

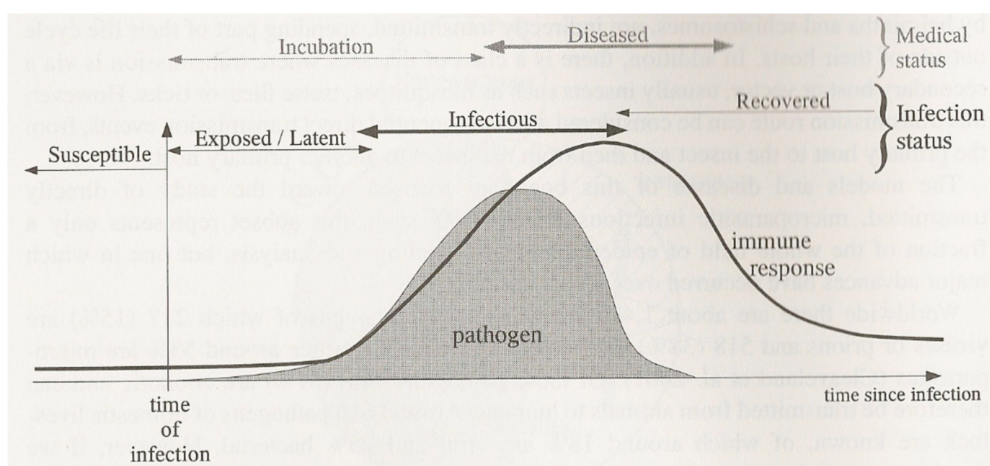
metapopulation, network & landscape models (the vast middle)

compartmental models (fully-mixed)

fine resolution; heterogeneous ⟷ coarse resolution; homogeneous

Compartmental models

- Assumptions:
  - population is well-mixed: all contacts equally likely
  - only need to keep track of number (or concentration) of hosts in different states or compartments
- Typical states
  - Susceptible: not exposed, not sick, can become infected
  - Infectious: capable of spreading disease
  - Recovered (or Removed): immune (or dead), not capable of spreading disease
  - Exposed: “infected”, but not infectious
  - Carrier: “infected” (although perhaps asymptomatic), and capable of spreading disease, but with a different probability



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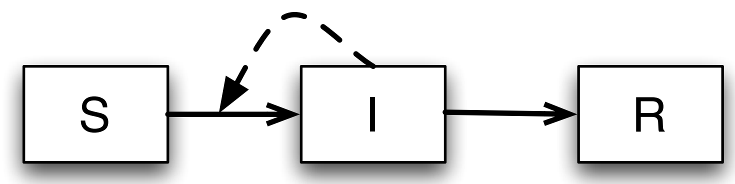
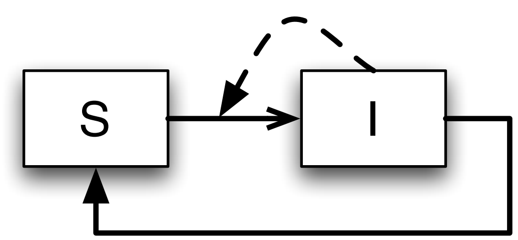
Compartmental models

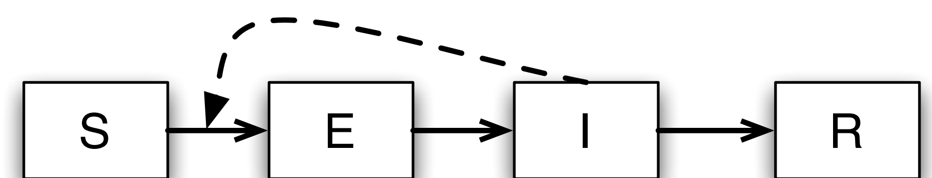
SIR: lifelong immunity



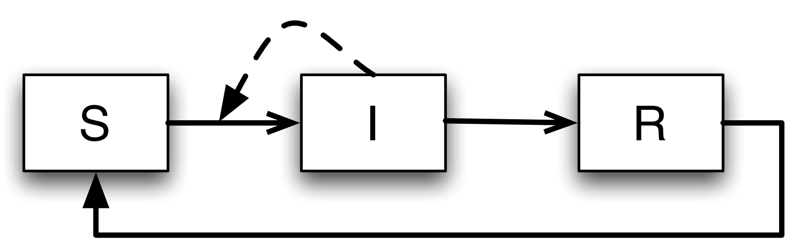
SIS: no immunity



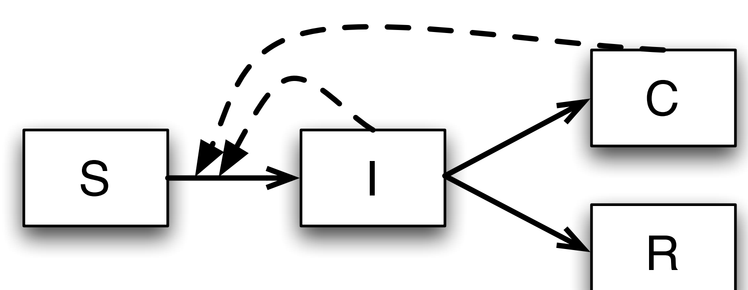
SEIR: SIR with latent (exposed) period



SIR with waning immunity

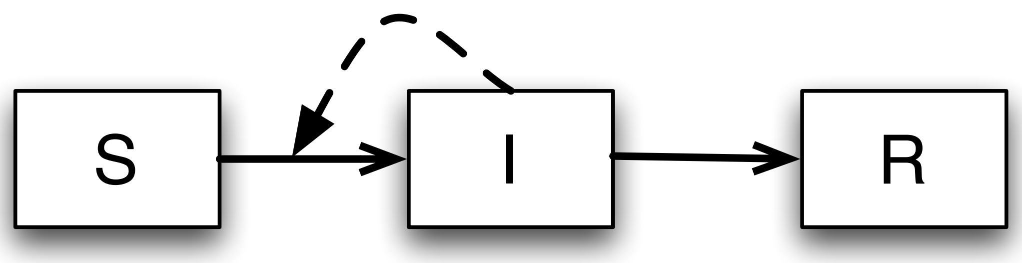


SIR with carrier state



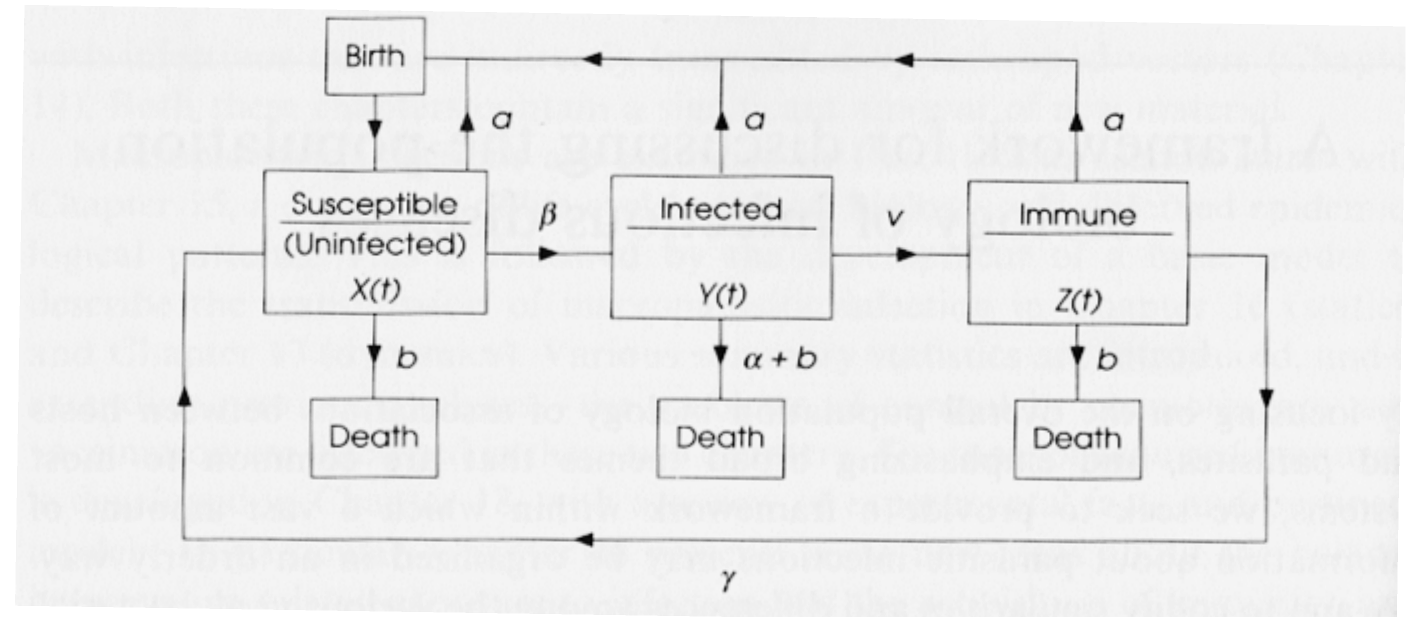
adapted from K&R

An aside on graphical notations

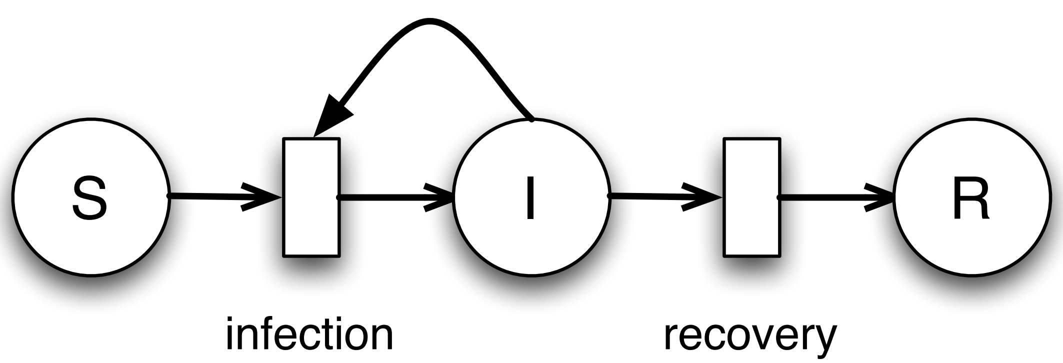


adapted from K&R

→ state transitions
- - → influence



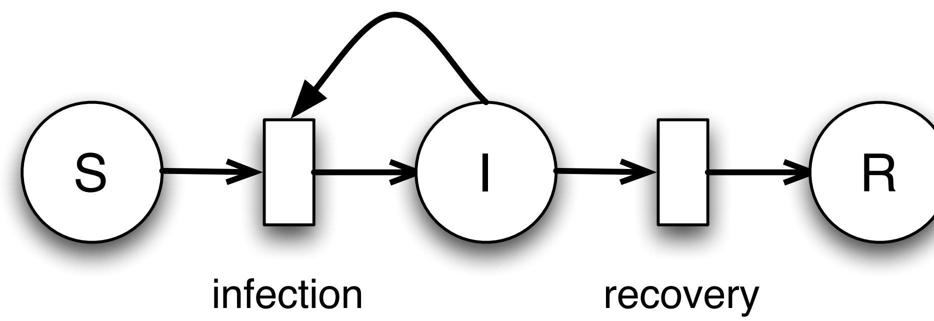
Anderson & May, Fig. 2.1



Petri Net: bipartite graph of places (states) and transitions (reactions)

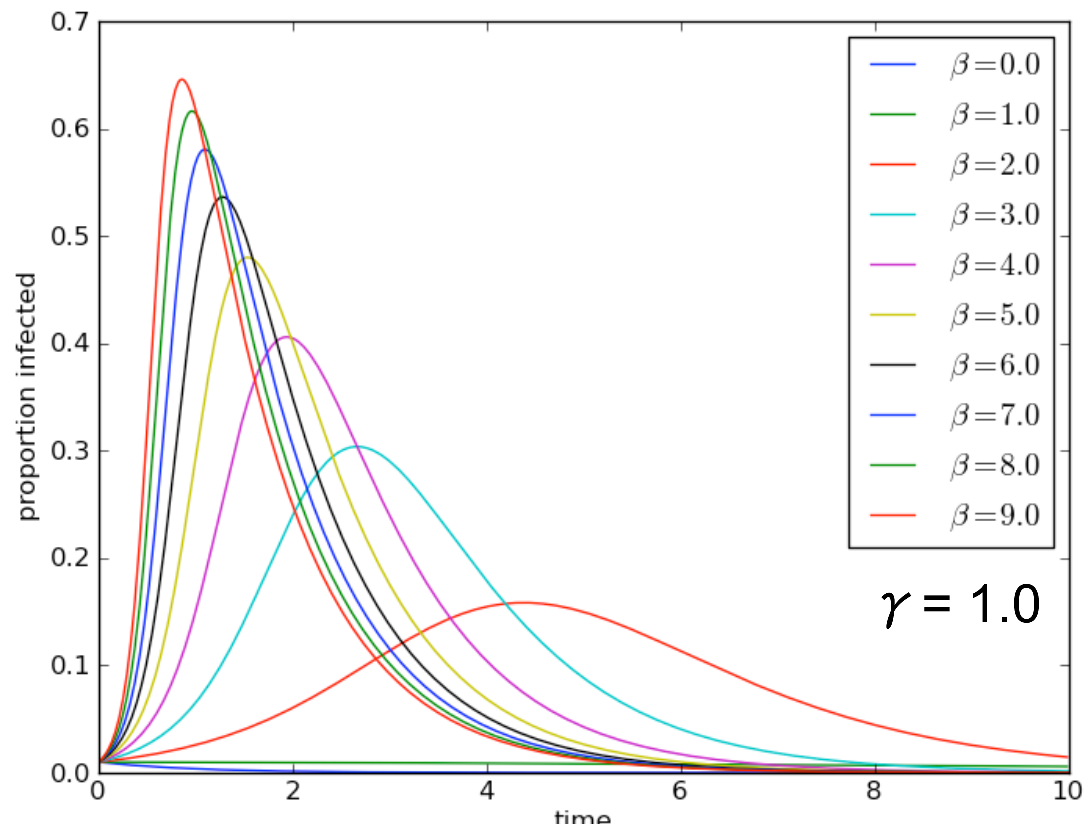
Susceptible-Infected-Recovered (SIR)

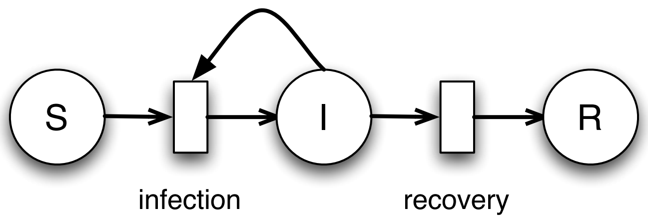
- Dates back to Kermack & McKendrick (1927), if not earlier
- Assume initially no demography
  - disease moving quickly through population of fixed size N
- Let:
  - X = # of susceptibles; proportion S = X/N
  - Y = # of infectives; proportion I = Y/N
  - Z = # of recovereds; proportion R = Z/N
  - note X+Y+Z = N, S+I+R=1
- average infectious period = $1/\gamma$
- force of infection λ
  - per capita rate at which susceptibles become infected



$$
\begin{aligned}
dS/dt &= -\beta SI \\
dI/dt &= \beta SI - \gamma I \\
dR/dt &= \gamma I
\end{aligned}
$$

SIR dynamics





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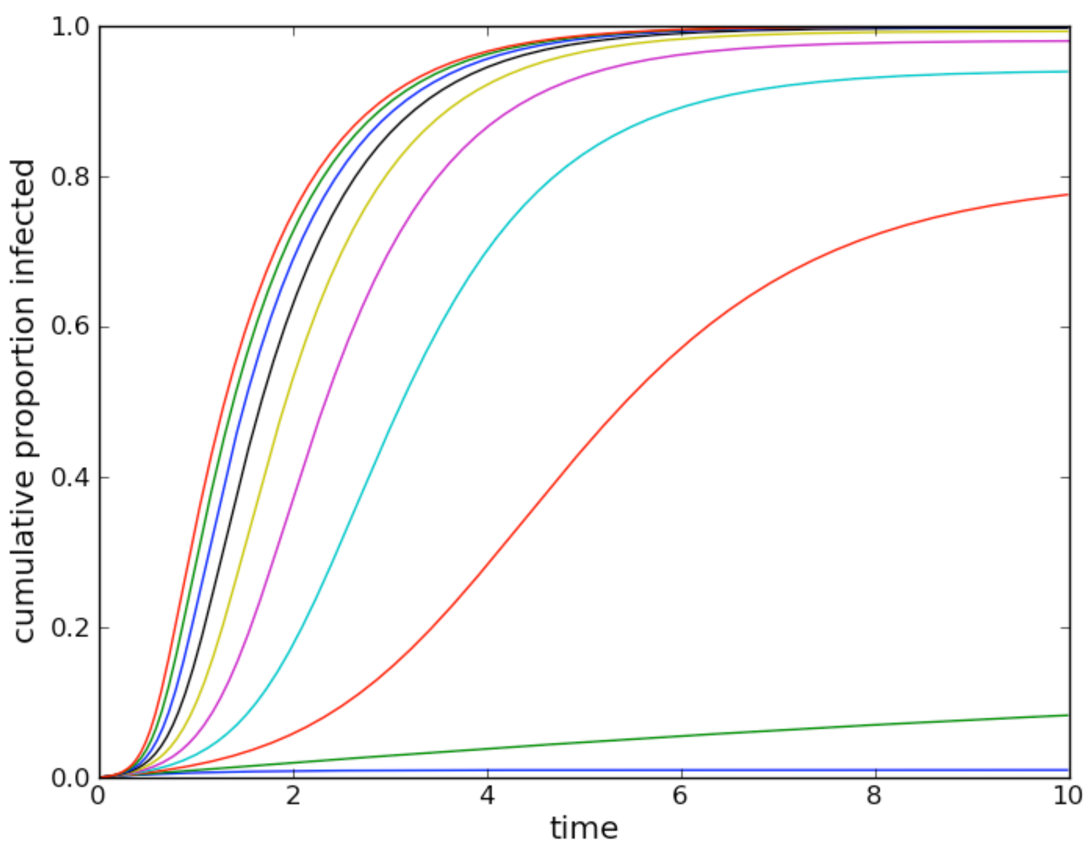
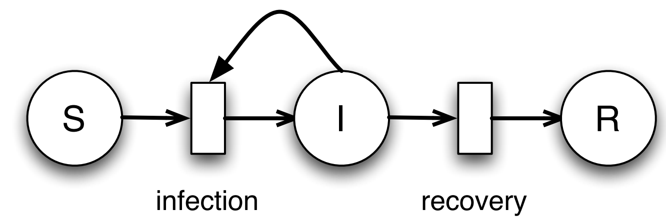


- Outbreak dies out if transmission rate is sufficiently low
- Outbreak takes off if transmission rate is sufficiently high

R_0 and the epidemic threshold

Introduction into fully susceptible population

$$\begin{aligned} dI/dt &= I(\beta - \gamma) \\ &> 0 \text{ if } \beta/\gamma > 1 \quad \text{(grows)} \\ &< 0 \text{ if } \beta/\gamma < 1 \quad \text{(dies out)} \end{aligned}$$



$$\begin{aligned} dS/dt &= -\beta SI \\ dI/dt &= \beta SI - \gamma I \\ dR/dt &= \gamma I \end{aligned}$$

$$\begin{aligned} dS/d\tau &= -R_0 SI \\ dI/d\tau &= R_0 SI - I \\ dR/d\tau &= I \\ \tau &= \gamma t \end{aligned}$$

- define *basic reproductive ratio*:

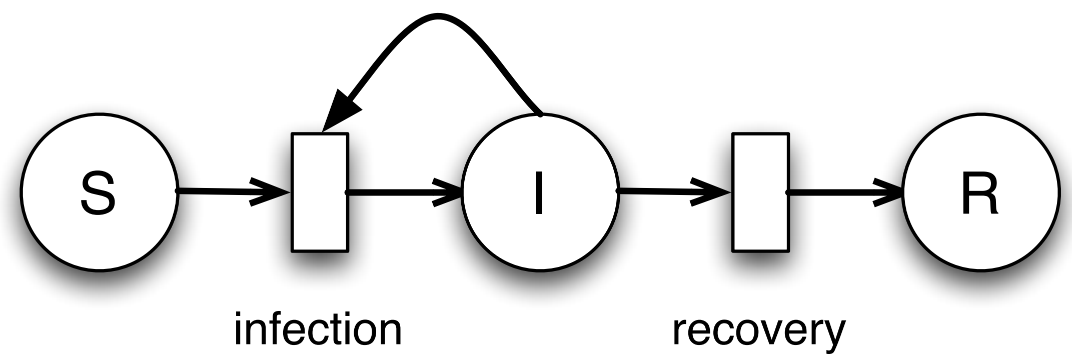
$$R_0 = \beta/\gamma$$

= average number of secondary cases arising from an average primary case in an entirely susceptible population

- epidemic threshold at $R_0 = 1$

Epidemic burnout

$$dS/dR = -\beta S/\gamma = -R_0 S$$



- integrate with respect to R:

$$S(t) = S(0)e^{-R(t)R_0}$$

$$R \leq 1 \implies S(t) \geq e^{-R_0} > 0$$

$$\begin{aligned} dS/dt &= -\beta SI \\ dI/dt &= \beta SI - \gamma I \\ dR/dt &= \gamma I \end{aligned}$$

- there will always be some susceptibles who escape infection

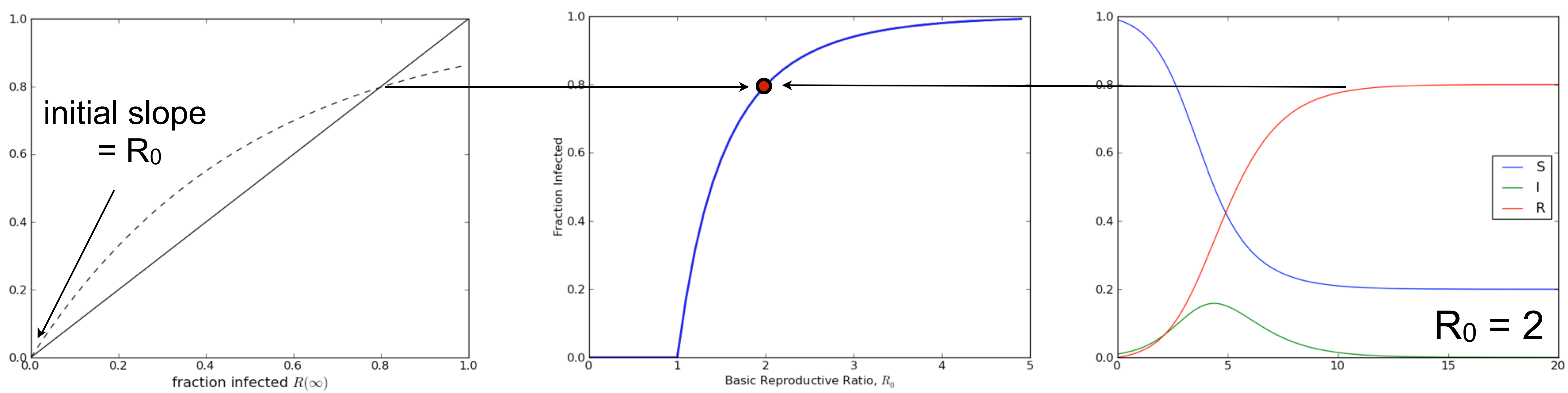
- the chain of transmission eventually breaks due to the decline in infecteds, not due to the lack of susceptibles

Fraction of population infected

$$S(t) = S(0)e^{-R(t)R_0}$$

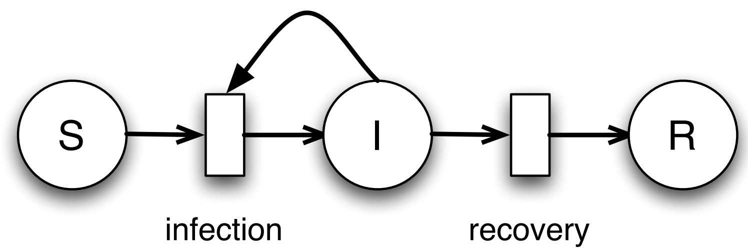
$$S(\infty) = 1 - R(\infty) = S(0)e^{-R(\infty)R_0}$$

- solve this equation (numerically) for R(∞) = total proportion of population infected



- outbreak: any sudden onset of infectious disease
- *epidemic*: outbreak involving non-zero fraction of population (in limit N→∞), or which is limited by the population size

Infectious disease module: SIR model



- Deterministic model
  - integrate ODE for (S,I,R) dynamics
  - epidemic threshold
  - size of outbreaks as a function of R0
- Stochastic model
  - simulate using Gillespie algorithm
  - outbreak size distributions
  - stochastic die-out

Beyond SIR

- Other compartmental models
  - SIS, SIRS, SIR+demography: endemic persistence
  - age-structured & risk-structured models
  - vectored diseases (transmitted by third-parties, e.g., mosquitoes)
  - temporally forced diseases: complex dynamics
  - control: vaccination, quarantine, culling, etc.

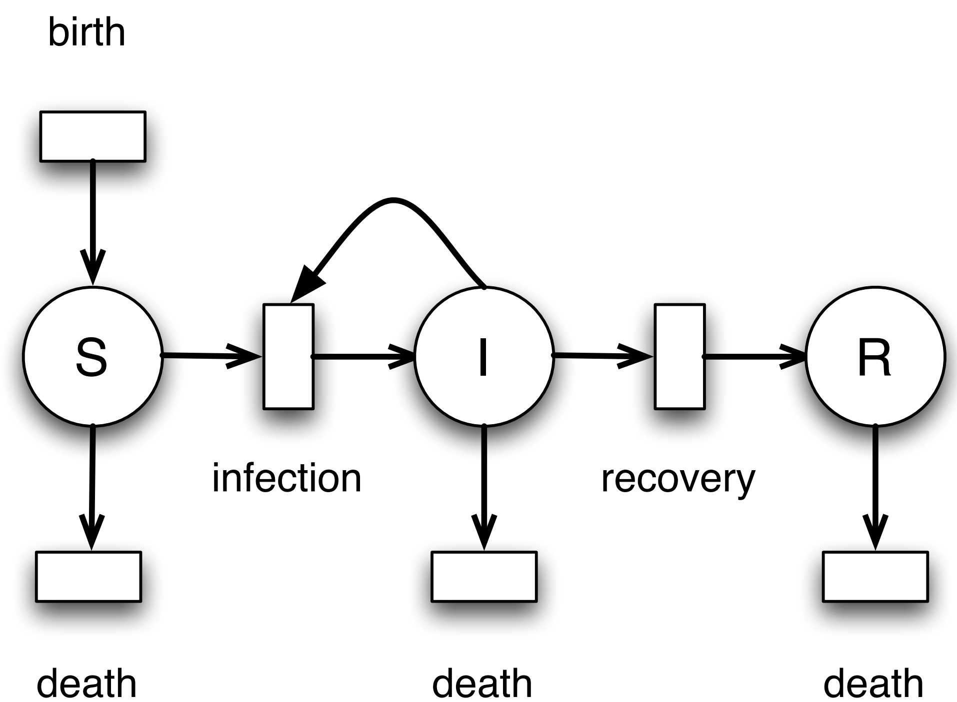
- Disease spread on networks
  - epidemic threshold ⟷ percolation transition
  - role of network topology, e.g., degree distribution

- Disease spread on metapopulations & landscape
  - roles of migration, dispersal and spillover
  - e.g., zoonotic diseases that cross from animals to humans

SIR with demography

- Allow for births and deaths
  - assume each happen at a constant rate μ
- R_0 reduced to account for both recovery and mortality

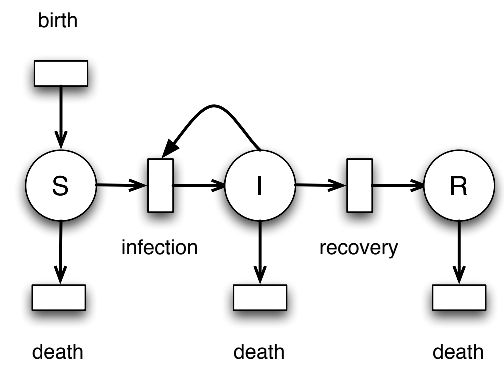
$$R_0 = \frac{\beta}{\gamma + \mu}$$



$$\begin{aligned} dS/dt &= \mu - \beta SI - \mu S \\ dI/dt &= \beta SI - \gamma I - \mu I \\ dR/dt &= \gamma I - \mu R \end{aligned}$$

Equilibria

$$dS/dt = dI/dt = dR/dt = 0$$



$$\begin{aligned} I \quad & (\beta S - (\gamma + \mu)) = 0 \implies \\ I \quad & = 0 \text{ or} \\ S \quad & = (\gamma + \mu)/\beta = 1/R_0 \end{aligned}$$

$$\begin{aligned} dS/dt &= \mu - \beta S I - \mu S \\ dI/dt &= \beta S I - \gamma I - \mu I \\ dR/dt &= \gamma I - \mu R \end{aligned}$$

- Disease-free equilibrium

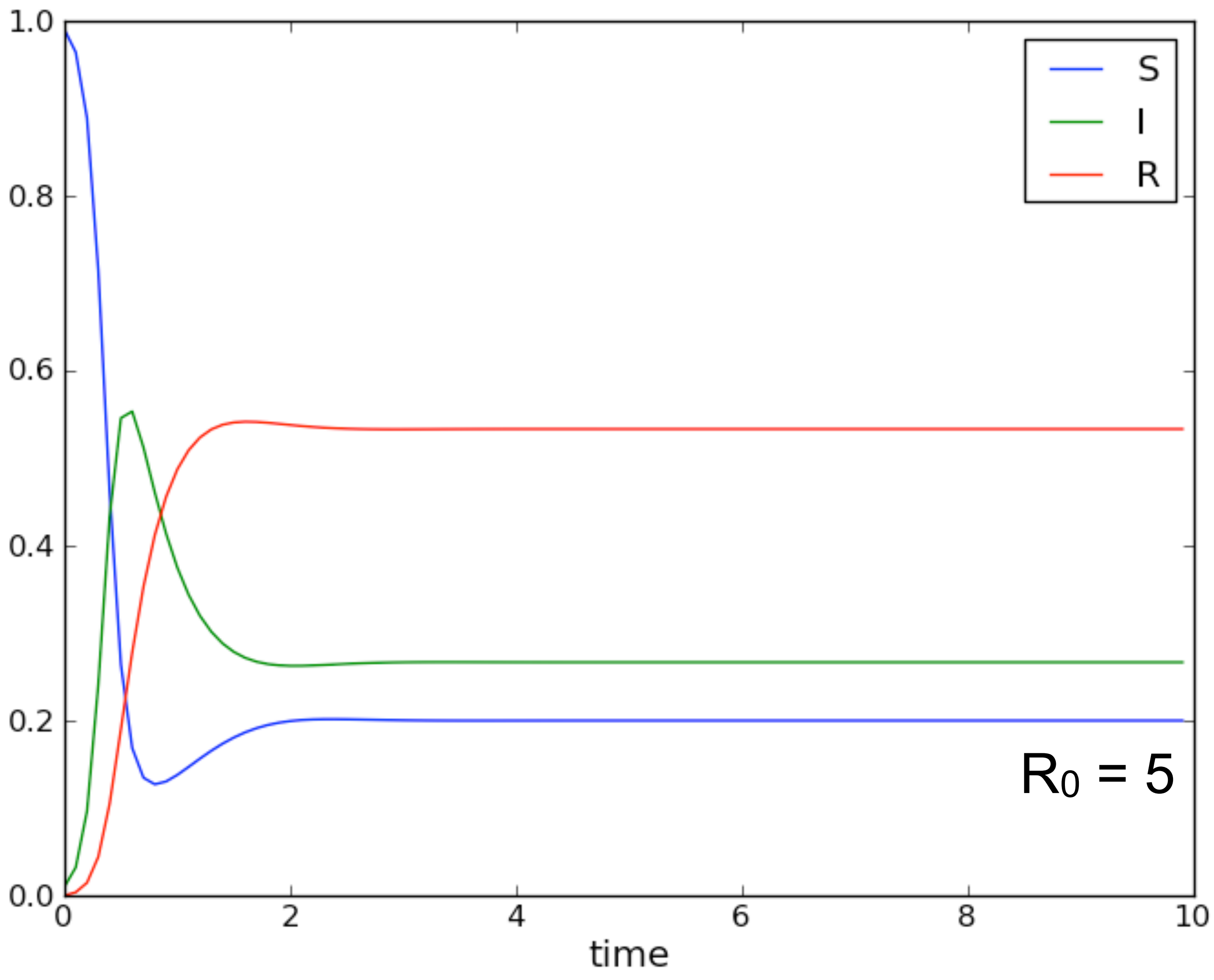
$$(S^*, I^*, R^*) = (1, 0, 0)$$

- Endemic equilibrium (only possible for R_0>1):

$$(S^*, I^*, R^*) = (\frac{1}{R_0}, \frac{\mu}{\beta}(R_0 - 1), 1 - \frac{1}{R_0} - \frac{\mu}{\beta}(R_0 - 1))$$

Endemic equilibrium

$$(S^*, I^*, R^*) = (\frac{1}{R_0}, \frac{\mu}{\beta}(R_0 - 1), 1 - \frac{1}{R_0} - \frac{\mu}{\beta}(R_0 - 1))$$



- Pool of fresh susceptibles enables infection to be sustained
- To establish equilibrium, must have each infective productive one new infective to replace itself
- $S = 1/R_0$

Vaccination

- minimum size of susceptible population needed to sustain epidemic

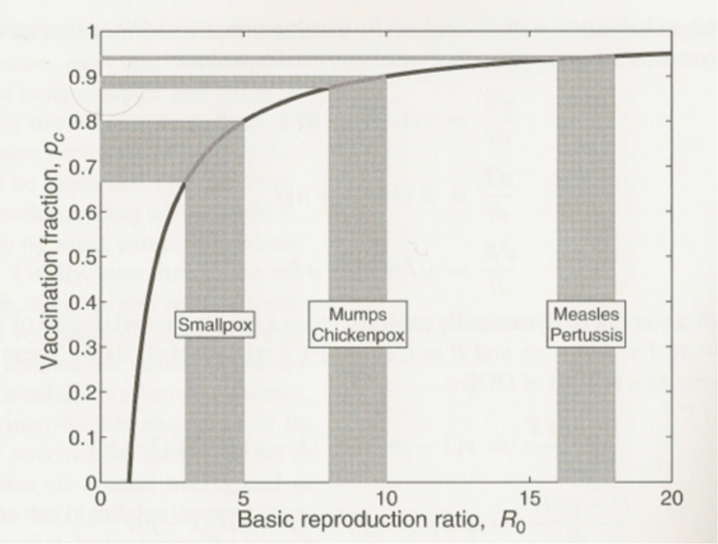
$$S_T = \gamma/\beta \implies R_0 = S/S_T$$

- vaccination reduces the size of the susceptible population
- immunizing a fraction p reduces R_0 to:

$$R_0^i = \frac{(1-p)S}{S_T} = (1-p)R_0$$

- critical vaccination fraction is that required to reduce $R_0 < 1$

$$p_c = 1 - \frac{1}{R_0}$$ "herd immunity"



K&R, Fig. 8.1

alternatively, p_c needed to drive endemic equilibrium to $I^*=0$:

$$(S^*, I^*, R^*) = (\frac{1}{R_0}, \frac{\mu}{\beta}(R_0 - 1), 1 - \frac{1}{R_0} - \frac{\mu}{\beta}(R_0 - 1))$$