
Exercises

8.7 Implementing Ising.¹ (Computation) ④

In this exercise, we will implement a simulation of the two-dimensional Ising model on a square lattice using the heat-bath and Metropolis algorithms. In the computer exercises portion of the web site for this book [129], you will find some hint files and graphic routines to facilitate working this exercise. The hints file should allow you to animate random square grids of ± 1 , giving you both the graphics interface and an example of random number generation.

The heat-bath algorithm flips spins one at a time, putting them into equilibrium with their neighbors: it is described in detail in Section 8.1.

(a) *Implement the heat-bath algorithm for the Ising model. When the temperature or external field is set, you should also reset the values in an array `heatBathProbUp[nUp]` storing the probability that a spin will be set to +1 given that `nUp` of its neigh-*

ors are currently pointing up (equal to +1). (Calculating these probabilities over and over again for millions of spin flips is unnecessary.) Explore the resulting behavior (say, as in Exercise 8.1).

The Metropolis algorithm also flips one spin at a time, but it always flips spins if the net energy decreases: it is described in detail in Exercise 8.6.

(b) *Implement the Metropolis algorithm for the Ising model. Here you will want to set up an array `MetropolisProbUp[s,nUp]` storing the probability that a spin which currently has value `s` will be set to +1 if `nUp` of its neighbors are currently up. Is Metropolis noticeably faster than the heat-bath algorithm?*

The Metropolis algorithm is always faster to equilibrate than the heat-bath algorithm, but is never a big improvement. Other algorithms can be qualitatively faster in certain circumstances (see Exercises 8.8 and 8.9).

¹From *Statistical Mechanics: Entropy, Order Parameters, and Complexity* by James P. Sethna, copyright Oxford University Press, 2007, page 176. A pdf of the text is available at pages.physics.cornell.edu/sethna/StatMech/ (select the picture of the text). Hyperlinks from this exercise into the text will work if the latter PDF is downloaded into the same directory/folder as this PDF.