Exercises

8.9 **Implementing Wolff.**[1] (Computation) ④

In this exercise, we will implement the Wolff algorithm of Exercise 8.8. In the computer exercises portion of the web site for this book [129], you will find some hint files and graphic routines to facilitate working this exercise.

Near the critical temperature T_c for a magnet, the equilibration becomes very sluggish: this is called *critical slowing-down.* This sluggish behavior is faithfully reproduced by the single-spin-flip heat-bath and Metropolis algorithms. If one is interested in equilibrium behavior, and not in dynamics, one can hope to use fancier algorithms that bypass this sluggishness, saving computer time.

(a) *Run the two-dimensional Ising model (either from the text web site or from your solution to Exercise 8.7) near $T_c = 2/\log(1+\sqrt{2})$ using a single-spin-flip algorithm. Start in a magnetized state, and watch the spins rearrange until roughly half are pointing up. Start at high temperatures, and watch the up- and down-spin regions grow slowly. Run a large enough system that you get tired of waiting for equilibration.*

The Wolff algorithm flips large clusters of spins at one time, largely bypassing the sluggishness near T_c. It can only be implemented at zero external field. It is described in detail in Exercise 8.8.

(b) *Implement the Wolff algorithm. A recursive implementation works only for small system sizes on most computers. Instead, put the spins that are destined to flip on a list* `toFlip`*. You will also need to keep track of the sign of the original triggering spin.*

```
While there are are spins toFlip,
  if the first spin remains parallel to the original,
      flip it, and
      for each neighbor of the flipped spin,
         if it is parallel to the original spin,
            add it to toFlip with probability p.
```

(c) *Estimate visually how many Wolff cluster flips it takes to reach the equilibrium state at T_c. Is Wolff faster than the single-spin-flip algorithms? How does it compare at high temperatures?*

(d) *Starting from a random configuration, change to a low temperature $T = 1$ and observe the equilibration using a single-spin flip algorithm. Compare with your Wolff algorithm.* (See also Exercise 12.3.) *Which reaches equilibrium faster? Is the dynamics changed qualitatively, though?*

[1] From *Statistical Mechanics: Entropy, Order Parameters, and Complexity* by James P. Sethna, copyright Oxford University Press, 2007, page 177. A pdf of the text is available at pages.physics.cornell.edu/sethna/StatMech/ (select the picture of the text). Hyperlinks from this exercise into the text will work if the latter PDF is downloaded into the same directory/folder as this PDF.